This version of David Jacobson's classic lens tutorial is from c. 1998. I've typeset the math and fixed a few spelling and grammatical errors; otherwise, it's as it was in 1997.

— Jeff Conrad 17 June 2017

Lens Tutorial

by David Jacobson for photo.net.

This note gives a tutorial on lenses and gives some common lens formulas. I attempted to make it between an FAQ (just simple facts) and a textbook. I generally give the starting point of an idea, and then skip to the results, leaving out all the algebra. If any part of it is too detailed, just skip ahead to the result and go on.

It is in 6 parts. The first gives formulas relating object (subject) and image distances and magnification, the second discusses *f*-stops, the third discusses depth of field, the fourth part discusses diffraction, the fifth part discusses the Modulation Transfer Function, and the sixth illumination. The sixth part is authored by John Bercovitz. Sometime in the future I will edit it to have all parts use consistent notation and format.

The theory is simplified to that for lenses with the same medium (e.g., air) front and rear: the theory for underwater or oil immersion lenses is a bit more complicated.

Object distance, image distance, and magnification

Throughout this article we use the word *object* to mean the thing of which an image is being made. It is loosely equivalent to the word "subject" as used by photographers.

In lens formulas it is convenient to measure distances from a set of points called *principal points*. There are two of them, one for the front of the lens and one for the rear, more properly called the primary principal point and the secondary principal point. While most lens formulas expect the object distance to be measured from the front principal point, most focusing scales are calibrated to read the distance from the object to the film plane. So you can't use the distance on your focusing scale in most calculations, unless you only need an approximate distance. Another interpretation of principal points is that a (probably virtual) object at the primary principal point formed by light entering from the front will appear from the rear to form a (probably virtual) erect image at the secondary principal point with magnification exactly one.

Nodal points are the two points such that a light ray entering the front of the lens and headed straight toward the front nodal point will emerge going straight away from the rear nodal point at exactly the same angle to the lens's axis as the entering ray had. The nodal points are identical to the principal points when the front and rear media are the same, e.g. air, so for most practical purposes the terms can be used interchangeably.

In simple double convex lenses the two principal points are somewhere inside the lens (actually $1/n$-th the way from the surface to the center, where n is the index of refraction), but in a complex lens they can be almost anywhere, including outside the lens, or with the rear principal point in front of the front principal point. In a lens with elements that are fixed relative to each other, the principal points are fixed relative to the glass. In zoom or internal focusing lenses the principal points generally move relative to the glass and each other when zooming or focusing.

When a camera lens is focused at infinity, the rear principal point is exactly one focal length in front of the film. To find the front principal point, take the lens off the camera and let light from a distant object pass through it “backwards.” Find the point where the image is formed, and measure toward the lens one focal length. With some lenses, particularly ultra wides, you can’t do this, since the image is not formed in front of the front element. (This all assumes that you know the focal length. I suppose you can trust the manufacturer’s numbers enough for educational purposes.)

S_o	object to front principal point distance
S_i	rear principal point to image distance
f	focal length
M	magnification

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$M = \frac{S_i}{S_o}$$

$$(S_o - f)(S_i - f) = f^2$$

$$M = \frac{f}{S_o - f} = \frac{S_i - f}{f}.$$

If we interpret $S_i - f$ as the “extension” of the lens beyond infinity focus, then we see that this extension is inversely proportional to a similar “extension” of the object.

For rays close to and nearly parallel to the axis (these are called *paraxial* rays) we can approximately model most lenses with just two planes perpendicular to the optic axis and located at the principal points. “Nearly parallel” means that for the angles involved, $\theta \approx \sin\theta \approx \tan\theta$. ($\approx$ means approximately equal.) These planes are called *principal planes*.

The light can be thought of as proceeding to the front principal plane, then jumping to a point in the rear principal plane exactly the same displacement from the axis and simultaneously being refracted (bent). The angle of refraction is proportional the distance from the center at which the ray strikes the plane and inversely proportional to the focal length of the lens. (The “front principal plane” is the one associated with the front of the lens. It could be behind the rear principal plane.)

Beyond the conditions of the paraxial approximation, the principal “planes” are not really planes but surfaces of rotation. For a lens that is free of coma (one of the classical aberrations) the principal planes, which could more appropriately be called equivalent refracting surfaces, are spherical sections centered around and object and image point for which the lens is designed.

Apertures, *f*-stop, bellows correction factor, pupil magnification

We define more symbols

D	diameter of the entrance pupil, i.e., diameter of the aperture as seen from the front of the lens
N	*f*-number (or *f*-stop) $D = f/N$, as in *f*/5.6
N_e	effective *f*-number, based on geometric factors, but not absorption

Light from an object point spreads out in a cone whose base is the entrance pupil. (The lens elements in front of the diaphragm form a virtual image of the physical aperture. This virtual image is called the *entrance pupil*.)

Analogous to the entrance pupil is the *exit pupil*, which is the virtual image of the aperture as seen through the rear elements.

Let us define N_e, the effective *f*-number, as

$$N_e = \frac{1}{2 \sin \theta_X},$$

where θ_X is the angle from the axis to the edge of the eXit pupil as viewed from the film plane. It can be shown that for any lens free of coma the following also holds

$$N_e = \frac{M}{2 \sin \theta_E}.$$

We will ignore the issue of coma throughout the rest of this document.

The first equation deals with rays converging to the image point, and is the basis for depth of field calculations. The second equation deals with light captured by the aperture, and is the basis for exposure calculations.

This section will explain the connection between N_e and light striking the film, relate this to N, and show to compute N_e for macro situations.

If an object radiated or reflected light uniformly in all directions, it is clear that the amount of light captured by the aperture would be proportional to the solid angle subtended by the aperture from the object point. In optical theory, however, it is common assume that the light follows Lambert's law, which says that the light intensity falls off with $\cos \theta$, where θ is the angle off the normal. With this assumption it can be shown that the light entering the aperture from a point near the axis is proportional to $\sin^2 \theta_E$, which is proportional to the aperture area for small θ_E.

If the magnification is M, the light from a tiny object patch of unit area gets spread out over an area M^2 on the film, and so the relative intensity on the film is inversely proportional to M^2. Thus the relative intensity on the film, RI, is given by

$$\text{RI} = \frac{\sin^2\theta_E}{M^2} = \frac{1}{4N_e^2},$$

with the second equality by the definition of N_e. (Of course the true intensity depends on the subject illumination, etc.)

For S_o very large with respect to f, M is approximately f/S_o and $\sin \theta_E$ is approximately $(D/2)/S_o$. Substituting these into the above formula, get that RI = $[(D/2)/f]^2 = 1/(4N^2)$, and thus for $S_o >> f$,

$$N_e = D/f = N.$$

For closer subjects, we need a more detailed analysis. We will take $D = f/N$ as determining D.

Let us go back to the original approximate formula for the relative intensity on the film, and substitute more carefully

$$\mathrm{RI} = \frac{\sin^2\theta_E}{M^2} = \frac{(D/2)^2}{(D/2)^2 + (S_o - z_E)^2}\frac{1}{M^2},$$

where z_E is the distance the entrance pupil is in front of the front principal point.

However, z_E is not convenient to measure. It is more convenient to use *pupil magnification*. The pupil magnification is the ratio of exit pupil diameter to the entrance pupil diameter:

$$p \qquad \text{pupil magnification} \left(\frac{\text{exit pupil diameter}}{\text{entrance pupil diameter}}\right).$$

For all symmetrical lenses and most normal lenses the aperture appears the same from front and rear, so $p \approx 1$. Wide angle lenses frequently have $p > 1$. It can be shown that $z_E = f(1-1/p)$, and substituting this into the above equation, carrying out some algebraic manipulations, and solving with $\mathrm{RI} = 1/(4\,N_e^2)$ yields

$$N_e = \sqrt{\left(\frac{M}{2}\right)^2 + \left[N\left(1+\frac{M}{p}\right)\right]^2}.$$

If we further assume θ_E is small enough that $\sin\theta_E \approx \tan\theta_E$, the $(M/2)^2$ term drops out and we get

$$N_e = N(1 + M/p).$$

This is the standard equation, and will be used throughout the rest of this document. The essence of the approximation is the distinction between the axial distance to the plane of the entrance pupil and the distance along the hypotenuse to the edge of the entrance pupil, which is the really correct form. Clearly in typical photographic situations that distinction is insignificant.

An alternative, but less fundamental, derivation is based on the relative illumination varying with the inverse square of the distance from the exit pupil to the film. This distance is just $f(1+M) - z_X$, where z_X is the distance the exit pupil is behind the rear principal point. It can be shown that $z_X = -f(p-1)$, so

$$\frac{N_e}{N} = \frac{f(1+M) + f(p-1)}{f + f(p-1)} = 1 + \frac{M}{p},$$

and hence $N_e = N(1+M/p)$.

It is convenient to think of the correction in terms of *f*-stops (powers of two). The correction in powers of two (stops) is $2 \log_2(1+M/p) = 6.64386 \log_{10}(1+M/p)$. Note that for most normal lenses $p = 1$, so the M/p can be replaced by just M in the above equations.

Circle of confusion, depth of field and hyperfocal distance.

The light from a single object point passing through the aperture is converged by the lens into a cone with its tip at the film (if the point is perfectly in focus) or slightly in front of or behind the film (if the object point is somewhat out of focus). In the out of focus case the point is rendered as a circle where the film cuts the converging cone or the diverging cone on the other side of the image point. This circle is called the circle of confusion. The farther the tip of the cone, i.e., the image point, is away from the film, the larger the circle of confusion.

Consider the situation of a “main object” that is perfectly in focus, and an “alternate object point” this is in front of or behind the main object.

S_{oa}	alternate object point to front principal point distance
S_{ia}	rear principal point to alternate image point distance
h	hyperfocal distance
C	diameter of circle of confusion
c	diameter of largest acceptable circle of confusion
N	*f*-stop (focal length divided by diameter of entrance pupil)
N_e	effective *f*-stop; $N_e = N\,(1+M/p)$
D	the aperture (entrance pupil) diameter; $D=f/N$
M	magnification; $M=f/(S_o - f)$

The diameter of the circle of confusion can be computed by similar triangles, and then solved in terms of the lens parameters and object distances. For a while let us assume unity pupil magnification, i.e., $p = 1$. When S_o is finite,

$$C = \frac{D\left(S_{ia} - S_i\right)}{S_{ia}} = \frac{f^2\left(S_o / S_{oa} - 1\right)}{N\left(S_o - f\right)}.$$

When S_o = infinity,

$$C = \frac{f^2}{N S_{oa}}.$$

Note that in this formula C is positive when the alternate image point is behind the film (i.e., the alternate object point is in front of the main object) and negative in the opposite case. In reality, the circle of confusion is always positive and has a diameter equal to $|C|$.

If the circle of confusion is small enough, given the magnification in printing or projection, the optical quality throughout the system, etc., the image will appear to be sharp. Although there is no one diameter that marks the boundary between fuzzy and clear, 0.03 mm is generally used in 35 mm work as the diameter of the acceptable circle of confusion. (I arrived at this by observing the depth of field scales or charts on/with a number of lenses from Nikon, Pentax, Sigma, and Zeiss. All but the Zeiss lens came out around 0.03 mm. The Zeiss lens appeared to be based on 0.025 mm.) Call this diameter c.

If the lens is focused at infinity (so the rear principal point to film distance equals the focal length), the distance to closest point that will be acceptably rendered is called the *hyperfocal distance*

$$h = \frac{f^2}{Nc}.$$

If the main object is at a finite distance, the closest alternative point that is acceptably rendered is at distance

$$S_{\text{close}} = \frac{h\,S_{\text{o}}}{h + (S_{\text{o}} - f)}$$

and the farthest alternative point that is acceptably rendered is at distance

$$S_{\text{far}} = \frac{hS_{\text{o}}}{h - (S_{\text{o}} - f)},$$

except that if the denominator is zero or negative, S_{far} = infinity.

We call $S_{\text{far}} - S_{\text{o}}$ the rear depth of field and $S_{\text{o}} - S_{\text{close}}$ the front depth field.

A form that is exact, even when $p \neq 1$, is

$$\text{Depth of field} = \frac{cN_{\text{e}}}{M^2\left[1 \pm (S_{\text{o}} - f)/h_1\right]} = \frac{cN(1 + M/p)}{M^2\left[1 \pm (Nc)/(fM)\right]}.$$

where $h_1 = f^2/(Nc)$, i.e., the hyperfocal distance given c, N, and f and assuming $p = 1$. Use '+' for front depth of field and '−' for rear depth of field. If the denominator goes zero or negative, the rear depth of field is infinity. ($\neq$ means "is not equal to.")

This is a very nice equation. It shows that for distances short with respect to the hyperfocal distance, the depth of field is very close to just cN_{e}/M^2. As the distance increases, the rear depth of field gets larger than the front depth of field. The rear depth of field is twice the front depth of field when $S_{\text{o}} - f$ is one third the hyperfocal distance. And when $S_{\text{o}} - f = h_1$, the rear depth of field extends to infinity.

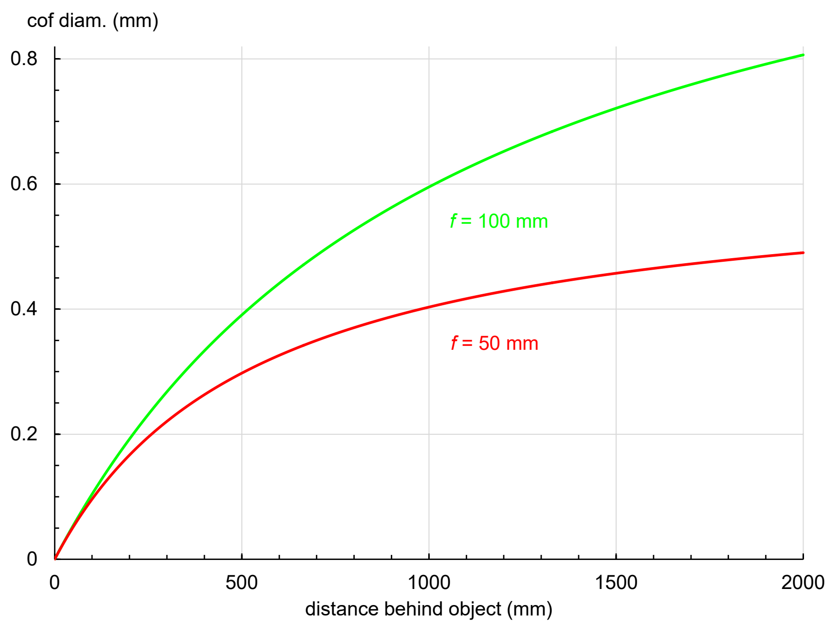
If we frame an object the same way with two different lenses, i.e., M is the same both situations, the shorter focal length lens will have less front depth of field and more rear depth of field at the same effective *f*-stop. (To a first approximation, the depth of field is the same in both cases.)

Another important consideration when choosing a lens focal length is how a distant background point will be rendered. Points at infinity are rendered as circles of size

$$C = \frac{fM}{N}.$$

So at constant object magnification a distant background point will be blurred in direct proportion to the focal length.

This is illustrated by the following example, in which lenses of 50 mm (red) and 100 mm (green) focal lengths are both set up to get a magnification of 1/10. Both lenses are set to *f*/8. The graph shows the circles of confusion as a function of the distance behind the object.



The slopes of both graphs are virtually identical out to well beyond where the diameter of the circle of confusion is 0.03 mm, showing that to a first approximation both lenses have the same depth of field. However, the size of the circle of confusion for in infinitely distant point is twice as large for the 100 mm lens (1.25 mm) as for the 50 mm lens (0.625 mm).

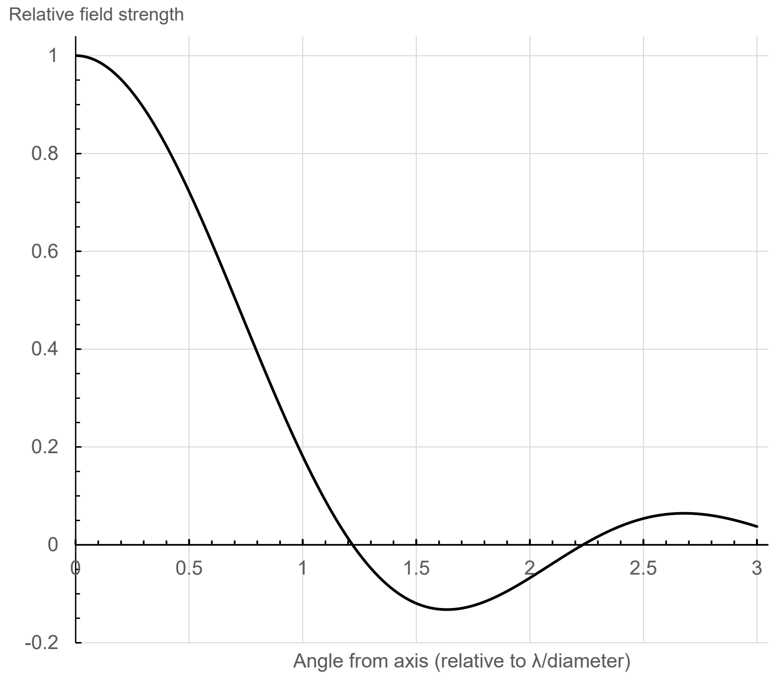
Diffraction

When a beam of parallel light passes through a circular aperture it spreads out a little, a phenomenon known as diffraction. The smaller the aperture, the more the spreading. The normalized field strength (of the electric or magnetic field) at angle ϕ from the axis is given by

$$2\,J_1(x)/x\,,$$

where $x = 2\phi\pi R/\lambda$, and where R is the radius of the aperture, λ is the wavelength of the light, and J_1 is the first order Bessel function. The normalization is relative to the field strength at the center. The power (intensity) is proportional to the square of this function.

The field strength function forms a bell-shaped curve, but unlike the classic e^{-x^2} one, it eventually oscillates about zero. Its first zero is at 1.21967 $\lambda/(2R)$. There are actually an infinite number of lobes after this, but about 83.8% of the power is in the circle bounded by the first zero.



Approximating the aperture-to-film distance as *f* and making use of the fact that the aperture has diameter *f* /*N*, it follows directly that the diameter of the first zero of the diffraction pattern is 2.43934 $N\lambda$. Applying this in a normal photographic situation is difficult, since the light contains a whole spectrum of colors. We really need to integrate over the visible spectrum. The eye has maximum sensitivity around 555 nm, in the yellow green. If, for simplicity, we take 555 nm as the wavelength, the diameter of the first zero, in mm, comes out to be 0.00135383 *N*.

As was mentioned above, the normally accepted circle of confusion for depth of field is .03 mm, but 0.03/0.00135383 = 22.1594, so we can see that at *f*/22 the diameter of the first zero of the diffraction pattern is as large is the acceptable circle of confusion.

A common way of rating the resolution of a lens is in line pairs per mm. It is hard to say when lines are resolvable, but suppose that we use a criterion that the center of a dark band receive no more than 80% of the light power striking the center of a light band. Then the resolution is $0.823/(\lambda N)$ lp/mm. If we again assume 555 nm, this comes out to 1482/*N* lp/mm, which is in close agreement with the widely used rule of thumb that the resolution is diffraction limited to 1500/*N* lp/mm. However, note that the MTF, discussed below, provides another view of this subject.

Modulation Transfer Function

The modulation transfer function is a measure of the extent to which a lens, film, etc., can reproduce detail in an image. It is the spatial analog of frequency response in an electrical system. The exact definitions of the modulation transfer function and of the related optical transfer function vary slightly amongst different authorities.

The 2-dimensional Fourier transform of the point spread function is known as the optical transfer function (OTF). The value of this function along any radius is the Fourier transform of the line spread function in the same direction. The modulation transfer function is the absolute value of the Fourier transform of the line spread function.

Equivalently, the modulation transfer function of a lens is the ratio of relative image contrast divided by relative object contrast of an object with sinusoidally varying brightness as a function of spatial frequency (e.g., cycles per mm). Relative contrast is defined as $(I_{max}-I_{min})/(I_{max}+I_{min})$. MTF can also be used for film, but since film has a non-linear characteristic curve, the density is first transformed back to the equivalent intensity by applying the inverse of the characteristic curve.

For a lens, the MTF can vary with almost every conceivable parameter, including *f*-stop, object distance, distance of the point from the center, direction of modulation, and spectral distribution of the light. The two standard directions are radial (also known as sagittal) and tangential.

The MTF for an ideal lens (ignoring the unavoidable effect of diffraction) is a constant 1 for spatial frequencies from 0 to infinity at every point and direction. For a practical lens it starts out near 1, and falls off with increasing spatial frequency, with the falloff being worse at the edges than at the center. Adjacency effects in film can make the MTF of film be greater than 1 in certain frequency ranges.

An advantage of the MTF as a measure of performance is that under some circumstances the MTF of the system is the product (frequency by frequency) of the properly scaled MTFs of its components. Such multiplication is always allowed when the phase of the waves is lost at each step. Thus it is legitimate to multiply lens and film MTFs or the MTFs of a two lens system with a diffuser in the middle. However, the MTFs of cascaded ordinary lenses can legitimately be multiplied only when a set of quite restrictive and technical conditions is satisfied.

As an example of some OTF/MTF functions, below are formulas for and plots of the OTFs three cases:

1. Diffraction for an *f*/22 aperture.
2. A 0.03 mm circle of confusion
3. The combination of these; more precisely, the OTF of an otherwise ideal lens with an *f*/22 aperture and defocused to produce a 0.03 mm circle of confusion.

Let λ be the wavelength of the light, and v the spatial frequency in cycles per mm. For diffraction the formula is

$$\begin{aligned}\mathrm{OTF}(\lambda, N, \nu) &= \frac{2}{\pi}\left[\arccos\left(\lambda N \nu\right) - \lambda N \nu \sqrt{1-\left(\lambda N \nu\right)^2}\right], \lambda N \nu \le 1 \\ &= 0, \lambda N \nu \ge 1\end{aligned}.$$

Note that for λ = 555 nm, the OTF is zero at spatial frequencies of $1801/N$ cycles per mm and beyond.

For a circle of confusion of diameter C,

$$\mathrm{OTF}\left(C,\nu\right) = 2J_1\left(\pi C\nu\right)/\left(\pi C\nu\right),$$

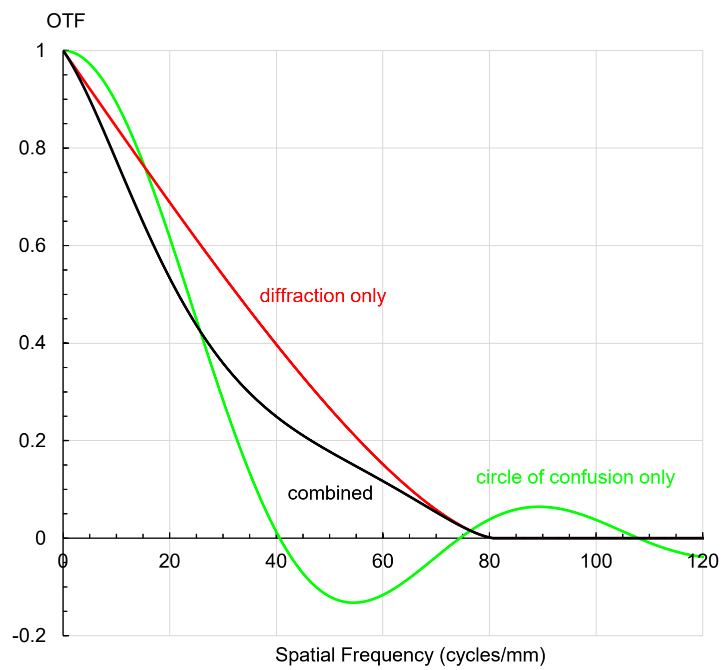
where, again, $J_1(x)$ is the first order Bessel function. The OTF goes negative at certain frequencies. Physically, this would mean that if the test pattern were lighter right on the optical center than nearby, the image would be darker right on the optical center than nearby. Some authorities use the term "spurious resolution" for spatial frequencies beyond the first zero. The MTF is the absolute value of the OTF.

Consider the case where there is a combination of diffraction and focus error dz, the distance the between the film plane and the plane of sharpest focusing. (A focus error of dz by itself would cause a circle of confusion of diameter dz/N.) The OTF for this combination is given by the following formula, which involves an integration that must be done numerically. Let $s = \lambda N v$, and $a = \pi v\, dz/N$. Then the OTF is given by

$$\begin{aligned} \mathrm{OTF} &= \frac{4}{\pi a}\int_{y=0}^{y=\sqrt{1-s^2}} \sin\left(a\sqrt{1-y^2} - s\right) dy,\ s<1 \\ &= 0,\ s \ge 1 \end{aligned}.$$

This formula is an approximation that is best at small apertures.

Here is a graph of the OTF of the *f*/22 diffraction limit, a 0.03 mm circle of confusion, and the combined effect:



Note how the combination is not the product of each of the effects taken separately.

Some authorities present MTF in a log-log plot.

The classic paper on the MTF for the combination of diffraction and focus error is by H.H. Hopkins, "The frequency response of a defocused optical system," Proceedings of the Royal Society A, v. 231, London (1955), pp 91–103. Reprinted in Lionel Baker (ed), *Optical Transfer Function: Foundation and Theory*, SPIE Optical Engineering Press, 1992, pp 143–153.

Illumination

(by John Bercovitz)

The Photometric System

Light flux, for the purposes of illumination engineering, is measured in lumens. A lumen of light, no matter what its wavelength (color), appears equally bright to the human eye. The human eye has a stronger response to some wavelengths of light than to other wavelengths. The strongest response for the light-adapted eye (when scene luminance $\geq$ 0.001 Lambert) comes at a wavelength of 555 nm. A light-adapted eye is said to be operating in the photopic region. A dark-adapted eye is operating in the scotopic region (scene luminance <= 10^{-8} Lambert). In between is the mesopic region. The peak response of the eye shifts from 555nm to 510 nm as scene luminance is decreased from the photopic region to the scotopic region. The standard lumen is approximately 1/680 of a watt of radiant energy at 555 nm. Standard values for other wavelengths are based on the photopic response curve and are given with two-place accuracy by the table below. The values are correct no matter what region you're operating in—they're based only on the photopic region. If you're operating in a different region, there are corrections to apply to obtain the eye's relative response, but this doesn't change the standard values given below.

Wavelength, nm	Lumens/watt
400	0.27
450	26
500	220
550	680
600	430
650	73
700	2.8

Following are the standard units used in photometry with their definitions and symbols.

Luminous flux, *F*, is measured in lumens.

Quantity of light, *Q*, is measured in lumen-hours or lumen-seconds. It is the time integral of luminous flux.

Luminous Intensity, *I*, is measured in candles, candlepower, or candela (all the same thing). It is a measure of how much flux is flowing through a solid angle. A lumen per steradian is a candle. There are 4π steradians to a complete solid angle. A unit area at unit distance from a point source covers a steradian. This follows from the fact that the surface area of a sphere is $4\pi r^2$.

Lamps are measured in MSCP, mean spherical candlepower. If you multiply MSCP by 4π, you have the lumen output of the lamp. In the case of an ordinary lamp which has a horizontal filament when it is burning base down, roughly 3 steradians are ineffectual: one is wiped out by interference from the base and two more are very low intensity since not much light comes off either end of the filament. So figure the MSCP should be multiplied by 4/3 to get the candles coming off perpendicular to the lamp filament. Incidentally, the number of lumens coming from an incandescent lamp varies approximately as the 3.6

power of the voltage. This can be really important if you are using a lamp of known candlepower to calibrate a photometer.

Illumination (illuminance), *E*, is the *areal density* of incident luminous flux: how many lumens per unit area. A lumen per square foot is a foot-candle; a one square foot area on the surface of a sphere of radius one foot and having a one candle point source centered in it would therefore have an illumination of one foot-candle due to the one lumen falling on it. If you substitute meter for foot you have a meter-candle or lux. In this case you still have the flux of one steradian but now it's spread out over one square meter. Multiply an illumination level in lux by 0.0929 to convert it to footcandles. $(\text{foot/meter})^2 = 0.0929$. A centimeter-candle is a phot. Illumination from a point source falls off as the square of the distance. So if you divide the intensity of a point source in candles by the distance from it in feet squared, you have the illumination in foot candles at that distance.

Luminance, *B*, is the *areal intensity* of an extended diffuse source or an extended diffuse reflector. If a perfectly diffuse, perfectly reflecting surface has one foot-candle (one lumen per square foot) of illumination falling on it, its luminance is one foot-Lambert or $1/\pi$ candles per square foot. The total amount of flux coming off this perfectly diffuse, perfectly reflecting surface is, of course, one lumen per square foot. Looking at it another way, if you have a one square foot diffuse source that has a luminance of one candle per square foot (π times as much intensity as in the previous example), then the total output of this source is pi lumens. If you travel out a good distance along the normal to the center of this one square foot surface, it will look like a point source with an intensity of one candle.

To contrast: Intensity in candles is for a point source while luminance in candles per square foot is for an extended source—luminance is intensity per unit area. If it's a perfectly diffuse but not perfectly reflecting surface, you have to multiply by the reflectance, *k*, to find the luminance.

Also to contrast: Illumination, *E*, is for the incident or incoming flux's areal *density*; luminance, *B*, is for reflected or outgoing flux's areal *intensity*.

Lambert's law says that a perfectly diffuse surface or extended source reflects or emits light according to a cosine law: the amount of flux emitted per unit surface area is proportional to the cosine of the angle between the direction in which the flux is being emitted and the normal to the emitting surface. (Note however, that there is no fundamental physics behind Lambert's "law." While assuming it to be true simplifies the theory, it is really only an empirical observation whose accuracy varies from surface to surface. Lambert's law can be taken as a definition of a perfectly diffuse surface.)

A consequence of Lambert's law is that no matter from what direction you look at a perfectly diffuse surface, the luminance on the basis of *projected* area is the same. So if you have a light meter looking at a perfectly diffuse surface, it doesn't matter what the angle between the axis of the light meter and the normal to the surface is as long as all the light meter can see is the surface: in any case the reading will be the same.

There are a number of luminance units, but they are in categories: two of the categories are those using English units and those using metric units. Another two categories are those which have the constant $1/\pi$ built into them and those that do not. The latter stems from the fact that the formula to calculate luminance (photometric brightness), *B*, from illumination (illuminance), *E*, contains the factor 1/π. To illustrate:

$B = (kE)(1/\pi)$
$B_{fl} = kE$

where:

B = luminance, candles/foot^2
B_{fl} = luminance, foot-Lamberts
k = reflectivity $0 < k < 1$
E = illuminance in foot-candles (lumens/foot^2)

Obviously, if you divide a luminance expressed in foot-Lamberts by π you then have the luminance expressed in candles /foot^2. ($B_{fl}/\pi = B$)

Other luminance units are:

stilb =	1 candle/square centimeter	sb
apostilb =	stilb/($\pi \times 10^4$) = 10^{-4} L	asb
nit =	1 candle/ square meter	nt
Lambert =	(1/π) candle/square cm	L

Below is a table of photometric units with short definitions.

Symbol	Term	Unit	Unit Definition
Q	light quantity	lumen-hour lumen-second	radiant energy as corrected for eye's spectral response
F	luminous flux	lumen	radiant energy flux as corrected for eye's spectral response
I	luminous intensity	candle	one lumen per steradian
		candela	one lumen per steradian
		candlepower	one lumen per steradian
E	illumination	foot-candle	lumen/foot^2
		lux	lumen/meter^2
		phot	lumen/centimeter^
B	luminance	candle/foot^2	see unit def's. above
		foot-Lambert	= (1/π) candles/foot^2
		Lambert	= (1/π) candles/centimeter^2
		stilb	= 1 candle/centimeter^2
		nit	= 1 candle/meter^2

Note: A lumen-second is sometimes known as a Talbot. To review:

Quantity of light, *Q*, is akin to a quantity of photons except that here the number of photons is pro-rated according to how bright they appear to the eye.

Luminous flux, *F*, is akin to the time rate of flow of photons except that the photons are pro-rated according to how bright they appear to the eye.

Luminous intensity, *I*, is the solid-angular density of luminous flux. Applies primarily to point sources. Illumination, *E*, is the areal density of incident luminous flux. Luminance, *B*, is the areal intensity of an extended source.

Photometry with a Photographic Light Meter

The first caveat to keep in mind is that the average unfiltered light meter doesn't have the same spectral sensitivity curve that the human eye does. Each type of sensor used has its own curve. Silicon blue cells aren't too bad. The overall sensitivity of a cell is usually measured with a 2856 K or 2870 K incandescent lamp. Less commonly it is measured with 6000 K sunlight.

The basis of using a light meter is the fact that a light meter uses the Additive Photographic Exposure System, the system which uses Exposure Values:

$$E_v = A_v + T_v = S_v + B_v$$

where:

E_v = Exposure Value
A_v = Aperture Value = $\lg_2 N^2$ where N = *f*-number
T_v = Time Value = $\lg_2 (1/t)$ where t = time in sec
S_v = Speed Value = $\lg_2 (0.3\, S)$ where S = ASA speed
B_v = Brightness Value = $\lg_2 B_{\text{fl}}$

$\lg_2$ is logarithm base 2 from which, for example:

$A_v(N = f/1) = 0$

$T_v(t = 1 \text{ sec}) = 0$

$S_v(S = \text{ASA } 3.125)$ E

$B_v(B_{\text{fl}} = 1 \text{ foot-Lambert}) = 0$

and therefore: $B_{\text{fl}} = 2^{Bv}$ E_v ($S_v = 0$) = B_v From the preceding two equations you can see that if you set the meter dial to an ASA speed of approximately 3.1 (same as $S_v = 0$), when you read a scene luminance level the E_v reading will be B_v from which you can calculate B_{fl}. If you don't have an ASA setting of 3.1 on your dial, just use ASA 100 and subtract 5 from the E_v reading to get B_v. (S_v @ ASA 100 = 5)

Image Illumination

If you know the object luminance (photometric brightness), the *f*-number of the lens, and the image magnification, you can calculate the image illumination. The image magnification is the quotient of any linear dimension in the image divided by the corresponding linear dimension on the object. It is, in the usual photographic case, a number less than one. The *f*-number is the *f*-number for the lens when focused at infinity—this is what's written on the lens. The formula that relates these quantities is given below:

$$E_{\text{image}} = \frac{t\pi B}{4N^2 (1+m)^2}$$

or:

$$E_{\text{image}} = \frac{tB_{\text{fl}}}{4N^2 (1+m)^2}$$

where:

E_{image}	is in foot-candles (divide by 0.0929 to get lux)
t	is the transmittance of the lens (usually 0.9 to 0.95 but lower for more surfaces in the lens or lack of anti-reflection coatings)
B	is the object luminance in candles/square foot
B_{fl}	is the object luminance in foot-Lamberts
N	is the *f*-number of the lens
M	is the image magnification

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This document is based on the Usenet Lens Tutorial, version 1.10 of October 26, 1996. The Usenet version and this version may diverge in the future.

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